# Game Theory Problems
Michał Krawczyk
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INDIVIDUAL DECISION MAKING

1. The table below represents Anne's preference over Caribbean holiday destinations. For example, she finds Aruba at least as attractive as Bonaire, $a \succcurlyeq b$, (there is a "1" in the first row, second column). Is this preference complete? reflexive? transitive? Can we call Anne a rational decision maker? Suppose this preference structure holds no matter what her wealth level is. Can you find a way in which a smart tour operator could keep extracting Anne's money from her? ("money pump") Derive the strict preference and indifference relations and represent them with appropriate tables.

| | Aruba | Bonaire | Curacao |
|---|---|---|---|
| Aruba | 1 | 1 | 0 |
| Bonnaire | 0 | 1 | 1 |
| Curacao | 1 | 0 | 1 |

2. An entrepreneur is considering three possible actions with respect to his business, yielding different returns in USD$10^6$ depending on market conditions, happening with prob. $p_l$, $p_m$, $p_h$ respectively, see the table.

| strategy; market demand | low | medium | high |
|---|---|---|---|
| invest | 1 | 16 | 49 |
| stay | 9 | 16 | 36 |
| withdraw | 16 | 16 | 16 |

a) Assume utility function $u(x) = x$. For each strategy specify the conditions under which it is optimal. Which decisions are rationalizable and why?

b) Suppose that all states of the world are equally likely. Which strategy would an expected value maximizer choose?

c) What about a decision maker with utility function $u(x) = x^{0.5}$?

3. Bill likes apples and oranges. He is always willing to exchange one orange for exactly two apples. Give three examples of utility functions that can represent this preference, two of which are non-linear in the number of apples or oranges that he consumes.

4. Claire can get from A to B by bike, by bus or by car. Traveling time depends on her choice of the mode of transportation and traffic conditions which she cannot affect. Going by bike takes 12 minutes, no matter what. Bus takes 12 minutes, plus extra 3 minutes if traffic is heavy. Going by car takes just 7 minutes in light traffic but twice that much in heavy traffic. Getting to B as fast as possible is Claire's only concern. Represent the situation using a decision matrix. Find out which decisions are strictly or weakly dominated (by which other decisions). Which decision is safest?

GAMES IN NORMAL FORM

5. In each of the following two-person games find

a) any dominant strategies

b) safest strategies

c) all Nash Equilibria

| | L | M | R |
|---|---|---|---|
| U | 0;2 | 2;0 | 3;6 |
| I | 4;1 | 0;2 | 2;0 |
| D | 1;2 | 3;0 | 5;1 |

| | L | M | R |
|---|---|---|---|
| U | 4;4 | 3;2 | 2;0 |
| I | 2;3 | 5;5 | -3;1 |
| D | -1;4 | 0;3 | 1;6 |

6. An employer hires an employee and promises him wage $w$. The employee can work ($W$) or shirk ($S$). Working is associated with cost of effort $e$, where $w > e > 0$. If the employee works, the employer obtains revenue $r$ and otherwise $\frac{r}{2}$. Employer cannot tell whether the employee is working or shirking unless she chooses to run inspection ($I$) at a cost of $c$. Inspection reveals whether the employee is working or not and in the latter case the wage is withheld. The decisions of the two players ($W/S$ and $I/NI$) are taken simultaneously.

   a) Represent this situation as a game in a matrix form (employee as row player)

   b) Under what condition(s) will this game have a pure strategy Nash Equilibrium?

   c) Assume the aforementioned condition(s) is (are) not satisfied. Derive the formulas for probability of working $p$ and probability of inspection $q$ in the mixed strategy Nash Equilibrium.

   d) Assume that $w = 5, e = 3, r = 10, c = 2$. Find the equilibrium strategies and outcomes (you can use previously obtained results).

   e) With the specific values as above, assume that employer can irreversibly commit to certain probability of inspection (i.e. she chooses $q$ first, it is made public and cannot be changed). Is it possible for the employer to obtain outcome higher than in the equilibrium determined above? What probability should she (approximately) choose? Comment on the difference with respect to the equilibrium outcome.

7. Adam and Barbara are eating out. Each of them has to simultaneously and independently choose the main dish (s)he is going to have. More expensive dishes are more tasty but none of them wants to appear too greedy. Plus, the bill needs to be paid. All of these complex considerations are captured by following utility functions ($p_A$ is the price of Adam's dish and $p_B$ is that of Barbara) they seek to maximize:

$$U_A(p_A, p_B) = (-p_A - p_B + 100)p_A$$

$$U_B(p_A, p_B) = (p_A + 20 - \frac{p_B}{2})p_B$$

Dishes come in prices ranging from PLN20 to PLN100

a) Derive the best response functions for Adam and Barbara.

b) Represent them graphically

c) Find all equilibria. Compute Adam's and Barbara's equilibrium payoffs.

d) Is there any combination of strategies that would yield higher payoffs to both? Is the equilibrium Pareto-efficient?

e) Find strategies that would maximize the sum of payoffs.

8. Location game (Osbourne/Rubinstein (OR)) Each of $n$ people chooses whether or not to become a political candidate, and if so which position to take. There is a continuum of citizens, each of whom has a favorite position; the distribution of favorite positions is given by a density function f on $[0; 1]$ with $f(x) > 0$ for all $x \in [0; 1]$. A candidate attracts the votes of those citizens whose favorite positions are closer to his position than to the position of any other candidate; if $k$ candidates choose the same position then each receives the fraction $\frac{1}{k}$ of the votes that the position attracts. The winner of the competition is the candidate who receives the most votes. Each person prefers to be the unique winning candidate than to tie for first place, prefers to tie for first place than to stay out of the competition, and prefers to stay out of the competition than to enter and lose.

   Find NE for $n = 2$. Show there are none for $n = 3$.

9. (Malawski)

   Two business partners independently and simulteneously decide on the level of effort they put in their joint project. With effort levels $e_1$ and $e_2$ the total gross proceeds are $P(e_1, e_2) = 4(e_1 + e_2 + \frac{e_1 e_2}{4})$, to be shared

equally by the partners. The cost associated with effort $e_i$ of any player $i$ is $e_i^2$. Assume that effort levels are real numbers on $[0, 4]$. Each player seeks to maximize his share of proceeds minus his individual cost.

a) Find NE of this game. What is each player's net gain?

b) What is the maximum possible net gain per player? What effort levels must be chosen to obtain it?

c) Show that strategies $e_1 = 0.5$ and $e_2 = 3$ are dominated.

10. Consider two firms competing in prices in a market for a homogeneous good. The firms simultaneously choose their prices, $p_1$ and $p_2$, from $[0, 100]$. All customers buy from the firm with the lowest price and if both firms set the same price, then each firm receives half of the market demand at that price. Both firms produce at a constant unit cost,

$c_1$ and $c_2$, respectively $0 \leq c_1 < c_2 \leq 100$, there are no fixed costs. Thus each firm maximizes $q_i(p_i - c_i)$, where $q_i$ represents the number of units it sells. Let market demand be $D(p) = 100 - p$.

a) For each firm, find its monopoly price; the price it would have set, had it been alone in the market.

b) Define the associated normal-form game–what strategies are available and how payoffs depend on the choice of strategies of both players? Are the payoff functions continuous ("smooth") in strategies, or can they change abrubptly when strategies change only slightly?

c) Do best replies always exist? Does there exist Nash equilibria in pure strategies? Does there exist any pure-strategy Nash equilibrium in undominated strategies? (distinguish two cases: $c_1 = c_2$ and $c_1 < c_2$)

d) Same questions if only integer prices $(0, 1, 2, ..., a)$ are allowed, assume that $c_1, c_2$ are integers as well.

Note: in the problems below, whenever necessary denote the probabilities with which Player 1 (row player) in her mixed strategy uses strategies 1, 2 by $p_1, 1 - p_1$ respectively (the case of two strategies) and similarly $p_1, p_2, 1 - p_1 - p_2$ (three strategies). Analogously for player 2 (column player) using $q_1$ and perhaps $q_2$.

11. Find all equilibria in pure and mixed strategies in the following games:

| | Left | Right |
|---|---|---|
| Up | 1,2 | 4,2 |
| Down | 2,4 | 1,3 |

| | Left | Right |
|---|---|---|
| Up | 4,2 | 1,3 |
| Down | 2,3 | 2,2 |

| | Left | Center | Right |
|---|---|---|---|
| Up | 1,2 | 4,2 | 4,1 |
| Down | 2,4 | 1,3 | 2,0 |

12. Air strike (OR) Army A has a single plane with which it can strike one of three possible targets. Army B has one anti-aircraft gun that can be assigned to one of the targets. The value of target $k$ is $v_k$, with $v_1 > v_2 > v_3 > 0$. Army A can destroy a target only if the target is undefended and A attacks it. Army A wishes to maximize the expected value of the damage and army B wishes to minimize it. Formulate the situation as a strategic game (matrix-form game) and find its mixed strategy Nash equilibria. (Hint: Note there are no pure strategy equilibria. Prove that in equilibrium player 1 will use strategy 1 with positive probability ($p_1 > 0$) And further, that if she uses strategy 3 with positive probability, she will also use strategy 2 with positive probability (if $p_3 > 0$ then also $p_2 > 0$). Note that $p_1 = 1$ cannot be a part of equilibrium. Hence look for two types of equilibria: $p_1 > 0, p_2 > 0, p_3 = 0$ and $p_1 > 0, p_2 > 0, p_3 > 0$.

13. (a tricky one) Consider a market with 2 firms which produce the same good. Firms 1 and 2 simultaneously choose price $p_1$ and $p_2$ from the set of nonnegative real numbers (thus: not only integers!). Assume that there are no costs of production. If $p_i < p_j$ then firm $i$ gets demand $D(p_i)$ and firm $j$ gets demand 0. The demand function takes value of $D(p) = \frac{1}{\sqrt{1+p}}$. If the firms charge equal prices, assume that demand is split evenly.

    a) Find the pure strategy Nash Equilibrium

    b) Find the symmetric mixed strategy Nash Equilibrium where players choose strategies from a distribution with full support on $[10, \infty)$

(means: players never choose anything below 10 but sometimes choose any number equal or higher than 10). HINT: What is the expected payoff of a player choosing $p = 10$? What is thus the expected payoff of a player choosing any other $p$? What is thus the probability of capturing the market with this price? Won't this now suffice to determine the distribution used by the other player – his mixed strategy? Check whether this is indeed a correct probability distribution.

14. Two rouges meet in a dark street. They both simultaneously choose to fight or yield. If they fight, the stronger will win. The winner obtains payoff of one, the loser the payoff of minus one. If only one wants to fight, he obtains the payoff of 1 and the other obtains 0. If none wants to fight, both get zero.

    a) Represent the situation by two matrices: first corresponding to the situation in which Player 2 is stronger than Player 1 and the second one corresponding to the situation in which Player 2 is weaker. Are these strictly competitive games? Find equilibria of each game.

    b) Now assume that Player 1 doesn't know whether 2 is stronger or not and assigns probability .4 to the event that he is stronger. Player 2 knows his type. Find NE of this Bayesian game.

    c) Same task, only change Player's 1 prior probability to .7.

15. Acquire a Company Game. Player 1 owns a company which is worth between $v$ to him. $v$ follows a uniform distribution between 0 and \$1m ("any number equally likely"). Player 1 knows $v$. Player 2 does not, but she knows she is a more able manager and thus in her hands the company would be worth $1.5v$. Player 2 makes a an offer–the price she is willing to pay, $p_2$. Simultaneously Player 1 decides on his minimum acceptable price. If $p_2 > p_1$, then Player 2 buys the company at *her* price ($p_2$), obtaining $1.5v$. Obviously, Player 1 obtains $p_2$ and loses the company worth $v$. If $p_2 \leq p_1$, both players get zero.

    a) What is the optimal strategy for Player 1 for any $v$ he observes?

    b) Given this strategy of Player 1, for any offer $p_2$ what is the distribution of value of the company, *conditional on the offer being accepted*?

c) What offer will be made by Player 1 in equilibrium?

AUCTIONS

16. Comparison of auction formats Three buyers compete in a sealed-bid auction. Their valuations of the object are known to be distributed independently and uniformly on $[0; 1]$. It so happened that their actual values are $v_1 = .7, v_2 = .9, v_3 = .2$, but everybody only knows her own value. Using the appropriate formulas from the lecture notes determine what bids will be submitted, who will win the auction at which price and what will be the seller's revenue if the auction mechanism is:

    a) first-price

    b) second-price

    c) all-pay

    Assume there is no reserve price and bidders are risk-neutral. Comment on your results in view of the Revenue Equivalence Theorem (but first read it carefully).

17. Third-price auction. Two identical objects are offered in an auction with $n > 2$ players. Every buyer is only interested in purchasing one object and everybody's value follows the same continuous distribution $F(v)$. No bidder cares who obtains the other object if she obtains one. Each buyer submits a single sealed bid. The rules of the auction stipulate that the objects shall be assigned to the two highest bidders, each of them paying the price equal to the third highest bid.

    a) Do buyers have dominant strategies?

    b) Can you name any equilibria in non-dominant strategies?

    Now assume that each buyer is interested in obtaining both objects. Specifically, $2n$ independent values following the distribution $F(v)$ are assigned to $n$ buyers (two per buyer). The higher of the two values observed by any player $i$, denoted by $v_{i1}$ is understood as the value of the first object she buys and the lower ($v_{i2}$) as the value of the second object she buys (recall that objects are indistinguishable, but players display decreasing marginal utility). Each player submits two

bids, objects are assigned to the players with highest bids, each of them paying the amount equal to the third highest bid. (If two highest bids have been submitted by the same player she obtains both objects and pays the price equal to the third highest bid per object).

c) Do buyers have dominant strategies now?

d) Should the higher bid submitted by any player $i$, denoted $b_{i1}$ be higher, equal or lower than $v_{i1}$? How about $b_{i2}$ and $v_{i2}$?

18. Consider a three-buyer auction with private values $v_1, v_2, v_3$. Each $v_i$ is drawn from a uniform distributions on $[\frac{1}{6}, \frac{1}{2}]$ but they are interdependent such that $v_1 + v_2 + v_3 = 1$. Try to find an auction format that yields revenue higher than second-price sealed-bid auction (consider equilibria in non-weakly-dominated strategies only). Verify your finding with appropriate calculations (or prove that there is no such auction format). **Hint: browse through the lecture notes!**

19. Three buyers compete in an auction. Each buyer $i (i = 1, 2, 3)$ observes her private signal $s_i$ only. The singals are drawn independently from a uniform distribution on $[0, 1]$ and the value of the object is $V = s_1 + s_2 + s_3$.

    a) Consider an ascending (English) auction. What is the maximum amount that a buyer can safely bid as long as nobody has dropped yet? Using arguments discussed in class, show that the buyer with lowest signal actually will drop at this amount in equilibrium. How will the other two update their beliefs about the value of the object? What is now the maximum amount that any of them can safely bid? Again, show that the buyer with intermediate signal will drop at this amount. Who will drop at which price and what will be the seller's revenue if actual signals are $s_1 = .6, s_2 = .8, s_3 = .4$

    b) What bids will the buyers with these signals submit in a second-price sealed-bid auction? What will be the seller's revenue?

20. A seller offers an object for sale. Valuations of two potential buyers, $v_1, v_2$, are identically and independently distributed; $v_i, i = 1, 2$ follows a uniform distribution on [0,1].

a) The seller employs a first-price sealed-bid auction mechanism. Assume that both bidders use linear strategies, i.e. $b_i(v_i) = a_i v_i, i = 1, 2$, where $a_i$ is a positive constant. Find the equilibrium bidding strategies (i.e. the values of $a_1, a_2$). NOTE: you may remember the outcome from the class but now prove that it is correct, using the assumption of linearity under $n = 2$.

b) Now assume that the seller uses a descending auction format: the price goes down from one and the first buyer to stop it buys the object at this price. Find the equilibrium bidding strategies.

c) Now assume that Buyer 2 is entitled to buy at the price offered by Buyer 1, i.e., Buyer 1 submits her bid of choice, say: $b$, and Buyer 1 either buys at this price or steps out (in which case Buyer 1 purchases the object at $b$). Find optimal strategies for both players.

d) Can you give other examples of Nash Equilibria of the Baysian games from point c)? HINT: they may involve non-credible threats.

e) Compare the expected revenues obtained by the seller under points a) and c). Does the Revenue Equivalence Theorem result hold? If not, why not?

21. Two buyers compete in a **second-price** sealed-bid auction. The object on sale is an oil field, consisting of two parts: I and II. The value of each part is independent from the value of the other and each of them is distributed uniformly on [0, 1]. The total value of the oil field is the same for any buyer, equal to the sum of the values of the two parts. Buyer 1 knows the value of Part I, call it $s_1$, but has no private information about the value of Part II. Buyer 2 knows the value of Part II, call it $s_2$, but has no private information about Part I.

a) Is it a common value or a private value auction?

b) What is players 1 expectation regarding the value of the entire field? (express it in terms of $s_1$) How about player 2 (in terms of $s_2$)?

c) Suppose each player bids this expectation. Can it happen that the winner of the auction loses money? (hint: try low values of $s_1$, $s_2$.)

d) Show that $b_i = 2s_i$, $i = 1, 2$ is an equilibrium in this game.

e) What will be the expected revenue for the seller? What will be expected profit for the buyers?

f) Is this the only *symmetric* equilibrium? Is this the only equilibrium in general? (hint: think of one player submitting a high bid no matter what and the other player submitting a low bid no matter what).

g) Are the strategies used in the equilibrium $b_i = 2s_i$, $i = 1, 2$ dominant strategies?

DYNAMIC GAMES

22. There are 6 tokens on the table. Two players alternate removing some of the tokens. In each move any player can either remove **exactly** one or **exactly** two tokens. Whoever removes the last token is the winner (tokens are worthless per se – it does not matter who removed how many of them).

    a) Represent this game as a game tree

    b) Which player is going to win? Find the subgame perfect Nash equilibrium using backward induction. Precisely describe full strategies.

23. (Levine) Consider a chain store game in which an entrant must decide whether or not to enter the market. If no entry occurs the incumbent monopolist gets 2 and the entrant nothing. If entry occurs, the incumbent must choose between fighting and cooperating. If he fights, both firms get 1. If he cooperates the firms split the market, each getting 1. Suppose that the game is repeated between a infinitely-lived incumbent with discount factor equal to d and a sequence of short-lived entrants. What is the extensive and normal form of the stage game? What is subgame perfect in the stage game? What other Nash equilibria are there in the normal form? What is wrong with the Nash equilibrium that is not subgame perfect? Why is enter/cooperate subgame perfect equilibrium of the repeated game? For what value of d is it a subgame perfect equilibrium for the incumbent to fight as long as he has fought in the past, for entrants not to enter as long as the incumbent has always fought, and for enter/cooperate to occur after an incumbent has cooperated at least once?

24. (Malawski) Adam and Barbara have to share 6 non-divisible cookies. They have agreed that Adam would distribute the cookies on two plates and Barbara would pick one of them.

    a) Draw the game tree and find the SPNE under the assumption that each kid only cares about the number of cookies (s)he gets (and prefers more to less). How many cookies will each of them get? Does the game have any other (non-perfect) equilibria? Explain why or why not.

    b) Now suppose there are three mutually indistinguishable Golden Chocolate Treasure Cookies (G) and three mutually indistinguishable Oatmeal Raisin Cookies (O). Draw a part of the game tree (at least four edges). How many strategies does Adam have now? Provide the complete description of SPNE strategies and resulting division of cookies if players utilities are given by:

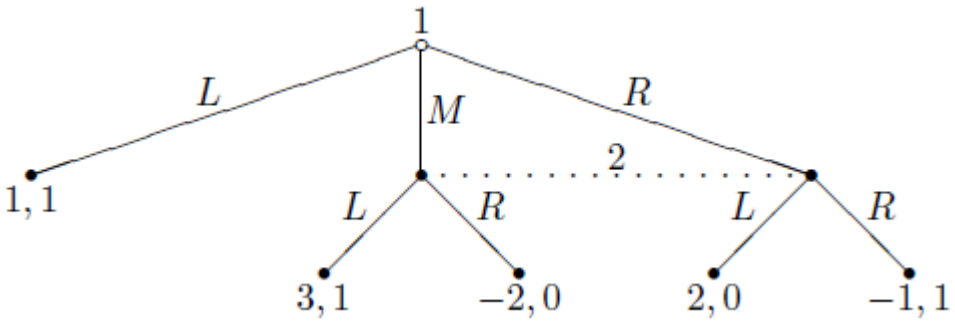
    $$u_A(G_A, O_A) = 2G_A + 8O_A, u_B(G_B, O_B) = 7G_B + 2O_B,$$

    Where $G_A$ is the number of GCT cookies that Adam receives and so on.

25. (Levine) Stephen J. Seagull must decide whether or not to start a new movie project. If he decides not to, he and Clod VandeCamp both get a utility of ten. If he decides to begin the project, both he and Clod must simultaneously decide who the director should be: George Spellbinder, or Ed Tree. If they disagree on the director, the movie isnt made, but both have wasted time, so they get only a utility of zero. If they agree on George Spellbinder, the movie will be a roaring success, and each gets a utility of 20. If they agree on Ed Tree, the movie will be terrible, and they will only get a utility of 5. Draw the extensive form of this game. Find the normal form. Find all the Nash equilibria. Find all the subgame perfect equilibria. Apply the concept of iterated elimination of weakly dominated strategies and the concept of forward induction.

26. (OR) Formulate the following parlor game as an extensive game with imperfect information. First player 1 receives a card that is either H or L with equal probabilities. Player 2 does not see the card. Player 1 may

Figure 1:




announce that her card is L, in which case she must pay \$1 to player 2, or may claim that her card is H, in which case player 2 may choose to concede or to insist on seeing player 1's card. If player 2 concedes, then he must pay \$1 to player 1. If he insists on seeing player 1's card then player 1 must pay him \$4 if her card is L and he must pay her \$4 if her card is H. Find the Nash equilibria of this game.

27. Find the set of sequential equilibria of the game in Figure 1

28. (Levine) President of the U.S. Bert C. Tree will shortly negotiating an agreement with the Premier of Hapistan. The Premier of Hapistan believes that President Tree is one of two types: sane (S) or crazy (C). The probability of type S is $p$. Prior to the negotiation, President Tree may bomb another country. The cost to the President of the bombing is zero for a crazy type and one for a sane type. After the bombing, the Premier may offer the President either a good deal (worth $G > 0$ to the President) or a bad deal (worth nothing to the President). The Premier gets one for offering the crazy type the good deal, one for offering the sane type the bad deal, and zero otherwise.

    a) Draw the extensive form.

    b) Find the pure strategy Nash equilibria of the game for the different values of p and G.

    c) Find a mixed strategy Nash equilibrium of the game for particular values $p = 3/4$ and $G = 2$.

d) From the cases above, give examples of a pooling, separating and hybrid equilibrium?

COOPERATIVE GAMES

29. There are four parties in the Interecclesial Committe of Ecumenia: Atheists, Buddhists, Christians and Druze, with 44, 24, 24 and 8 votes respectively. Any coalition with more than 66 votes is worth 1 and otherwise it is worth 0. Find the Shapley Value. (hint: to make it less tedious, you can first guess what will the unfortunate Druze folks' Shapley Value be (explain why!)).

30. In a symultanous symmetric three-player game each player has three options: Neutral (N), Sacrifice (S) and Malicious (M). If all play Neutral, the payoffs are (0,0,0). If any player plays S, she decreases her own payoff by 1 and increases payoffs of others by 2. If any player plays M, she decreases everybody's payoff by 1. These effect are cumulative. E.g. action profile $(N, S, M)$ yields payoffs $(0+2-1, 0-1-1, 0+2-1) = (1, -2, 1)$ (whereby for each player we have considered the effects of actions taken by player 1, then 2, then 3.).

    a) Represent this game in a matrix form (three matrices, each corresponding to one action by, say, player 3).

    b) Transform this game into a coalitional game by calculating the characteristic function, that is, for each coalition calculate the joint payoff that they can guarantee (i.e. what they will get if others are trying to hurt them).

    c) Find the core of the resulting game (Hint: do not forget about objections of one-person coalitions, note that the game is not necessarily normalized to $v(1) = v(2) = v(3) = 0$)

    d) Find the Shapley Value (Hint: do you need any calculations?)

31. Three players simultaneously choose Cooperate (C) or Defect (D). Payoff or every cooperator is equal to the total number of cooperators. Payoff of every defector is equal to the number of cooperators plus 1.1.

a) Represent this situation as a three-player non-cooperative game. Use two matrices, one corresponding to player 3 cooperating, the other to player 3 defecting, with Player 1 as row player and Player 2 as column player. Remember to display payoffs to all players in the matrix. Find all pure-strategy Nash equilibria.

b) Turn this game into a coalitional game (determine what each coalition can jointly surely get no matter what others do).

c) Identify the core of the coalitional game.

32. Consider three firms operating in an oligopolistic market. Each of them simultaneously makes an independent choice of the level of production, subject to its capacity constraint: $q_1 \leq 4, q_2 \leq 14, q_3 \leq 14$. There are no production costs. The resulting market price is $p = 40 - q_1 - q_2 - q_3$. Firms aim at maximum profits.

a) Represent this situation as a non-cooperative game.

b) Turn this game into a coalitional game

c) Identify the core of the coalitional game

d) Now, consider payoff profile induced by the Nash equilibrium of the non-coop game. Which inequalities defining the core do you expect to be satisfied? Which ones do you expect to be violated? About which ones you cannot tell before doing the calculations? Why?

e) Verify your claims from point d): calculate the NE of the non-coop game and check which inequalities are satisfied.

33. Consider three firms operating in an oligopolistic market. Each of them simultaneously makes an independent choice of price, choosing either $p_i = 2$ (cheap) or $p_i = 3$ (expensive). There are no production costs. There are 12 customers, each buying one unit. If all firms charge the same price, each attracts four customers. If one firm is cheap and two are expensive, the cheap one attracts six customers, and expensive firms attract three customers each. If two firms are cheap, they attract five customers each and the expensive firm attracts two customers. Firms aim at maximum profits, i.e. maximize product of own price and number of customers.

a) Represent this situation as a three-player non-cooperative game. Use two matrices, one corresponding to the choice of $p_3 = 2$, the other to $p_3 = 3$, with Firm 1 as row player and Firm 2 as column player. Find all pure-strategy Nash equilibria.

b) Turn this game into a coalitional game (determine what each coalition can jointly surely get no matter what others do).

c) Identify the core of the coalitional game

d) Now, for each pure strategy Nash equilibrium consider payoff profile induced by it. Which inequalities defining the core are satisfied, which are violated? Comment on your results.